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MINIMAL AUTOMATON FOR A FACTORIAL, TRANSITIVE, AND RATIONAL LANGUAGE

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Abstract. We prove that every FTR-language R (factorial, transitive, rational) admits a minimal irreducible deterministic automaton. This automaton can be computed in three different ways. First, it admits an intrinsic definition depending only on the syntactic monoid of the language R. Secondly, it is a quotient for a simple equivalence relation of every irreducible deterministic automaton recognizing R. Thirdly, it is a subautomaton of the classical minimal deterministic automaton associated with R, computable with a simple algorithm.

1. Introduction

The notion of FTR-languages appears in the theory of symbolic systems: in this theory, they are named transitive sofic systems [3]; and the automata used to recognize them, called Fisher automata, are irreducible deterministic automata.

In a previous paper [1], we studied a related notion: the notion of a rational cover [2]. Rational covers are recognized by trimmed automata, and FTR-languages are a subclass of the class of rational covers. An FTR-language is a rational cover without singular factor. In [1], we proved the existence of a minimal trimmed automaton for a rational cover. In this paper, we prove a more precise result. In the same time, we correct some points from [1] which were partly inaccurate. In fact, in the case of rational covers with singular factor, the application of some trimmed automaton on the minimal one is not exactly a morphism (there is a mistake in the proof).

So, it appears that the concept which provides interesting and clear results with respect to the notion of minimality is the FTR-language one. It is what we do here. Nevertheless, it remains to consider the whole class of languages recognized by trimmed automata (factorial bi-extendable, rational languages), not only the class of rational covers, and to look for some properties of minimality.

2. Preliminaries

Let A be a finite alphabet. A^* denotes the free monoid of finite words upon A. The empty word is denoted by 1 and $A^+ = A^* - \{1\}$.

Let $u \in A^*$; u' is a *factor* of u iff $\exists v, w \in A^*$ $u = vu'w$. The set of factors of u is $F(u)$.

2.1. Automata

A *finite automaton* is a quadruple, $\mathscr{A} = (Q, F, I, T)$ such that

- Q is a finite set of states,
- $F \subset Q \times A \times Q$ is the set of transitions,
- $I \subset Q$ is the set of initial states,
- $T \subset Q$ is the set of final states.

$\mathscr{A}^*$, the set of finite words recognized by $\mathscr{A}$, is the set of labels of the paths which start in some state of I and stop in some state of T.

- $\mathscr{A}$ is *deterministic* iff

$$(q, a, q') \in F \quad \text{and} \quad (q, a, q'') \in F \Rightarrow q' = q'';$$

and, in that case, for every word $u \in A^*$ and every state $q \in Q$, $q \cdot u$ denotes (if it exists) the single state q' such that there exists a path from q to q' labelled by u.

- $\mathscr{A}$ is *complete* iff

$$\forall a \in A \, \forall q \in Q \, \exists q' \in Q \quad (q, a, q') \in F.$$

- $\mathscr{A}$ is *strongly connected* iff for every pair $(q, q') \in Q \times Q$, there exists a path from q to q'.
- $\mathscr{A}$ is *irreducible* iff it is strongly connected and $I = T = Q$.

An *irreducible deterministic automaton* will be denoted by *ida*. $\mathscr{B} = (Q', F', I', T')$ is a *subautomaton* of $\mathscr{A}$ iff

$$Q' \subset Q, \qquad F' = F \cap (Q' \times A \times Q'), \qquad I' = I \cap Q', \qquad T' = T \cap Q'.$$

2.2. Languages

A language $R \subset A^*$ is said to be

- *rational* iff it is recognized by a finite automaton,
- *transitive* iff $\forall u, v \in R \, \exists w \in A^*$ $uwv \in R$,
- *factorial* iff $\forall u \in R$ $F(u) \subset R$.

A language satisfying these three properties is called an *FTR-language*.

Proposition 2.1 (Fischer [5]). *Every FTR-language is recognized by an irreducible automaton and conversely.*

2.3. Morphisms of ida's

The usual definition of morphisms of automata [4] applied to an ida is the following one: Let $\mathscr{A} = (Q, F, Q, Q)$ and $\mathscr{B} = (Q', F', Q', Q')$ be two ida's. A partial function $\varphi : Q \to Q'$ is a *morphism* from A to B iff

- φ is a surjective function,
- $\forall a \in A \, \forall q \in Q$ $(q\varphi) \cdot a \subset (q \cdot a)\varphi$.

One can remark that if φ is a morphism from A to B then $\mathscr{B}^* \subset \mathscr{A}^*$ [4].

2.4. Transition monoid-syntactic monoid

Let $\mathscr{A}=(Q, F, I, T)$ be a finite deterministic automaton. Let M be the monoid of partial functions from Q to Q provided with the operation of composition of functions. Let $\varphi_{\mathscr{A}}$ be the morphism of monoids defined by $\forall a \in A$ $a\varphi_{\mathscr{A}}$ is the partial function from Q to Q such that $\forall q \in Q$ $q(a\varphi_{\mathscr{A}}) = q \cdot a$. The *transition monoid* M_t of $\mathscr{A}$ is the image of A^* by $\varphi_{\mathscr{A}}$: $M_t = A^*\varphi_{\mathscr{A}}$. Let L be a part of a monoid N. The syntactic congruence modulo L is the congruence in N defined by

$$u \sim_L v \quad \text{iff} \quad \forall x, y \in N \; xuy \in L \Leftrightarrow xvy \in L.$$

The *syntactic monoid* of L is the quotient monoid $N_{/\sim_L}$.

2.5. Green's relations and 0-minimal ideal of a monoid (cf. [6, 7])

Let M be a monoid. Five equivalence relations R, L, H, J, D are usually defined in M, named Green's relations:

- $aRb \Leftrightarrow aM = bM$,
- $aLb \Leftrightarrow Ma = Mb$,
- $aJb \Leftrightarrow MaM = MbM$,
- $aHb \Leftrightarrow aRb$ and aLb,
- $D = R \vee L$.

If $a \in M$, L_a (resp. R_a) denotes the L-class (resp. R-class) of a. We recall here some classical results [7] we shall use later.

Proposition 2.2. *If M is a finite monoid, then $D = J$.*

A D-class is usually represented with an eggs-box where every cell represents an H-class, every arrow an R-class and every column an L-class. The main result is the following.

Proposition 2.3 (Pin [7]). *M is a finite monoid. Let a, $b \in M$ such that aRb. There exist u, $v \in M$ such that $au = b$ and $bv = u$. Let ρ_u and ρ_v be the right translations. Then ρ_u and ρ_v are inverse bijections from L_a onto L_b and from L_b onto L_a respectively. These bijections preserve the H-classes, i.e.:*

$$\forall x, y \in L_a \ (\textit{resp. } L_b) \quad xHy \; \Leftrightarrow \; x\rho_u H y\rho_u \ (\textit{resp. } x\rho_v H y\rho_v).$$

There is of course, a dual property if we suppose aLb. A D-class is shown in Fig. 1.

A *left* (resp. *right*) *ideal* is a part $I \subset M$ such that $MI = I$ (resp. $IM = I$). An ideal is a part $I \subset M$ such that $MIM = I$. A monoid M has a zero if there exists $0 \in M$ such that $M0 = 0M = 0$. If M has a zero, a (resp. left, right) ideal is 0-minimal if it is not reduced to 0, and minimal for inclusion among the (resp. left, right) ideals not reduced to 0.

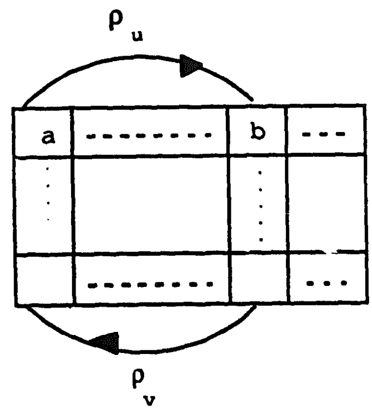


Fig. 1.

Proposition 2.4 (Pin [7]). *If I is a (resp. left, right) 0-minimal ideal, then for every $a \in I-\{0\}$, $I-\{0\}$ is the J (resp. L, R)-class of a.*

3. Minimal ida of an FTR-language

We can now give the main result of this paper.

Theorem 3.1. *Let R be an FTR-language. There exists a minimal ida $\mathscr{B}_0$ which recognizes R in the following sense: $\mathscr{B}_0^* = R$ and for every ida $\mathscr{A}$ such that $\mathscr{A}^* = R$, there exists a morphism $\varphi: \mathscr{A} \to \mathscr{B}_0$.*

Proof. (1) First, let us suppose that $R \neq A^*$. Let $\mathscr{A} = (Q, F, Q, Q)$ be an ida a recognizing R. $\mathscr{A}$ is not complete because $R \neq A^*$. Let $s \in Q$ be a hole state. $\mathscr{A}_c = (Q \cup \{s\}, F_c)$ is the complete automaton we get by adding to R the state s and the following transitions:

$$\forall a \in A \quad (s, a, s) \in F_c,$$

$$\forall q \in Q\ \forall a \in A \quad (q, a, s) \in F_c \text{ iff } (Q \times \{a\} \times Q) \cap F = \emptyset.$$

Let M_t be the transition monoid of $\mathscr{A}_c$. Then, the canonical morphism $\theta: A^* \to M_t$ has the following properties [3]:

- M_t has a zero which is $s\theta$ and $R = (M_t - \{0\})\theta^{-1}$,
- M_t has a unique 0-minimal ideal T.

So, we define an ida $\mathscr{B}$ such that $\mathscr{B}^* = R$ and a morphism φ from $\mathscr{B}$ to $\mathscr{A}$ in the following way: $T-\{0\}$ is a D-class and for every $x \in T-\{0\}$, $I = R_x \cup \{0\}$ is a 0-minimal right ideal. So, the states of $\mathscr{B}$ are the elements of $I-\{0\}$, for a fixed x. Transitions of $\mathscr{B}$ are defined by

$$\forall r \in I-\{0\}\ \forall a \in A \quad r \cdot a = r(a\theta) \text{ if } r(a\theta) \neq 0.$$

At last, let $r_0 \in I-\{0\}$. There exists a state $q_0 \in Q$ such that $q_0 r_0 \neq s$. The morphism φ is defined by

$$\forall r \in I-\{0\} \quad r\varphi = q_0 r.$$

One can remark that $q_0 r \neq s$. Indeed, r and r_0 are in the same R-class R_x. So, there exists r' such that $rr' = r_0$. Now suppose $q_0 r = s$. Then, $q_0 r_0 = q_0 rr' = sr' = s$, which is a contradiction, so $q_0 r \neq s$. It remains only to prove that $\mathscr{B}$ is an ida, $\mathscr{B}^* = R$ and that φ is a morphism from $\mathscr{B}$ to $\mathscr{A}$. $I-\{0\}$ is an R-class, so it implies that $\mathscr{B}$ is strongly connected.

(1) The function φ is surjective iff $\forall q \in Q\ \exists r \in I-\{0\}\ q_0 r = q$. We know that $q_0 r_0 \neq s$, so there exists $r_1 \in M_t$ such that $q_0 r_0 r_1 - q$, because $\mathscr{A}$ is irreducible. So, $r_0 r_1 \neq 0$ and $r_0 r_1 \in I-\{0\}$. Thus, $q = (r_0 r_1)\varphi$.

(2) $\forall r \in I-\{0\}\ \forall a \in A\ \ (r\varphi)\cdot a = (q_0 r)\cdot a = q_0 r(a\theta)$ and $(r\cdot a)\varphi = (r(a\theta))\varphi = q_0 r(a\theta)$. So φ is a morphism, and $\mathscr{A}^* \subset \mathscr{B}^*$.

Now, let $u \in \mathscr{B}^*$: $\exists r, r' \in I-\{0\}\ r' = r\cdot u$. So, $r'\varphi = q_0 r' = q_0(r\cdot u) = q_0 r(u\theta)$. But $q_0 r = q \neq s$, so $q\cdot u = q_0 r(u\theta) = q_0 r' = r'\varphi \neq s$. And u is the label of a path in $\mathscr{A}$ starting from $q = q_0 r$. So $u \in \mathscr{A}^*$. Let M_s the syntactic monoid of $M_t-\{0\}$, that is, the quotient of M_t by the congruence $\sim_{M_t-\{0\}}$ and ψ the canonical morphism $M_t \to M_s$. Clearly, for every $u, v \in A^*$, we have

$$u \sim_R v \Leftrightarrow u\theta \sim_{M_t-\{0\}} v\theta \Leftrightarrow u\theta\psi = v\theta\psi.$$

So, modulo an isomorphism, M_s is the syntactic monoid of R. Consequently, it is independent of the choice of the automaton $\mathscr{A}$. We can remark that 0φ is a zero in M_s and that $0\psi\{-1\} = 0$. On the other hand, $I\psi$ is a 0-minimal right ideal of M_s. So, ψ defines a morphism from the automaton $\mathscr{B}$ to the automaton $\mathscr{B}_0$ whose set of states is $I\psi-\{0\}$ and whose transitions are

$$\forall r \in I\psi-\{0\}\ \forall a \in A \quad r\cdot a = r(a\theta\psi) \text{ if } r(a\theta\psi) \neq 0.$$

It is clear that $\psi : \mathscr{B} \to \mathscr{B}_0$ is a morphism and $\mathscr{B}_0^* = \mathscr{B}^* = R$. $\mathscr{B}_0$ is obviously an ida. We stress that $\mathscr{B}_0$ depends only on R because all the 0-minimal right ideals of M_s are isomorphic. Now, we have the scheme in Fig. 2. And it suffices to remark that ψ can be factorized by φ iff

$$(*) \qquad \forall r, r' \in I-\{0\} \quad r\varphi = r'\varphi \Rightarrow r\psi = r'\psi.$$

Implication $(*)$ is proved using Proposition 2.3. Indeed, $r\varphi = r'\varphi \Leftrightarrow q_0 r = q_0 r'$. Let us suppose that $urv \neq 0$. Then, we shall prove that $ur'v \neq 0$, which implies

$$\begin{tikzcd} \mathscr{B} \arrow[r, "\varphi"] \arrow[d, "\psi"'] & \mathscr{A} \arrow[dl, dashed, "?"] \\ \mathscr{B}_0 & \end{tikzcd}$$

Fig. 2.

$r\psi = r'\psi$. If $r'v = 0$ then we would have $q_0r'v = q_0rv = s$. But, there exists v' such that $r = rvv'$ because $rM_t = rvM_t$. So, $q_0r = q_0rvv' = sv' = s$. But, $q_0r \neq s$ and there is a contradiction. So $r'v \neq 0$ and $tv \in I - \{0\}$.

On the other hand, $ur \neq 0$, so $ur \in L_r$. By Proposition 2.3, $ur'v$ belongs to the L-class of $r'v$ and so $ur'v \neq 0$. Now, since $r\varphi = r'\varphi \Rightarrow r\psi = r'\psi$, there exists a surjective function $\hat{\psi}$ of the set of states of $\mathscr{A}$ onto $\mathscr{B}_0$ such that $\varphi\hat{\psi} = \psi$. And it is easy to verify that $\hat{\psi}$ is a morphism from $\mathscr{A}$ to $\mathscr{B}_0$. $\hat{\psi}$ is defined by $\forall q \in Q\ q\hat{\psi} = r\psi$ where $r \in q\varphi - \{-1\}$ (cf. Fig. 3). So, we have built an ida $\mathscr{B}_0$ depending only on R such that $\mathscr{B}_0^* = R$, and a morphism $\hat{\psi}$ from $\mathscr{A}$ to $\mathscr{B}_0$.

(2) It remains to consider the case when $R = \mathscr{A}^*$. Let $\mathscr{A} = (Q, F, Q, Q)$ an ida such that $\mathscr{A}^* = R$. If $\mathscr{A}$ is complete, let $\mathscr{B}_0$ be the complete automaton which has only one state. Then, $\mathscr{B}_0^* = A^*$ and clearly, there is a morphism from $\mathscr{A}$ to $\mathscr{B}_0$. If $\mathscr{A}$ is not complete, we come back to the first construction by adding a zero to M_t. Let $M_t' = M_t \cup \{0\}$. Then, $R = (M_t' - \{0\})\theta^{-1}$ and the 0-minimal ideal of M_t' is the minimal ideal of M_t. At this point, the proof is identical to the first case and $\mathscr{B}_0$ is a complete automaton with a single state because the minimal ideal of the syntactic monoid of A^* has only one element. Actually, the following corollary proves that the last case cannot be done. □

$$\begin{tikzcd}
 & \mathscr{B} \arrow[dl, "\varphi"'] \arrow[dr, "\psi"] & \\
\mathscr{A} \arrow[rr, "\hat{\psi}"] & & \mathscr{B}_0
\end{tikzcd}$$

Fig. 3.

Corollary 3.2. *If $\mathscr{A}$ is an ida such that $\mathscr{A}^* = A^*$, then $\mathscr{A}$ is complete.*

Proof. There exists a morphism $\hat{\psi}$ from $\mathscr{A}$ to the complete automaton $\mathscr{B}_0$ with a single state q_0 of $\mathscr{A}$ according to the previous proof. So, for every state q of $\mathscr{A}$ and every $a \in A$, $(q\hat{\psi}) \cdot a \subset (q \cdot a)\hat{\psi}$. But $q\hat{\psi} = q_0$ and $q_0 \cdot a = q_0$, so $q \cdot a$ is not empty and $\mathscr{A}$ is complete. □

We illustrate this first result with an example.

Example 3.3. Let R the FTR-language recognized by the ida $\mathscr{A}$ in Fig. 4. θ is the canonical morphism $A^* \to M_t$ with $a\theta = \alpha$, $b\theta = \beta$, $c\theta = \gamma$. The 0-minimal ideal of

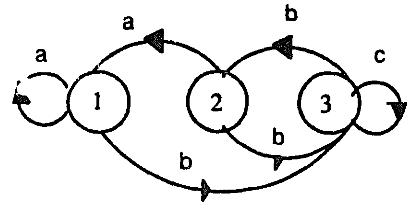


Fig. 4.

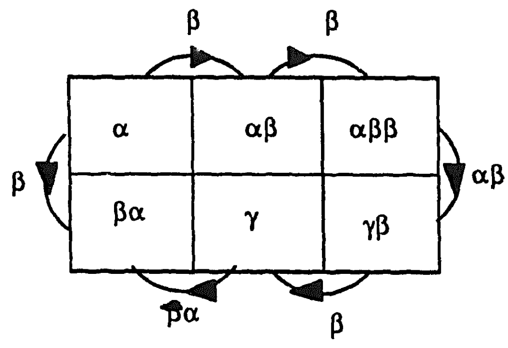
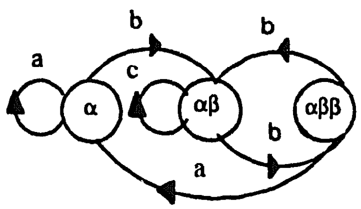


Fig. 5.



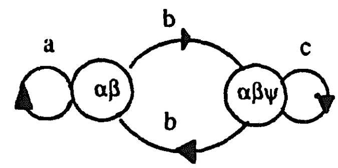


Fig. 7.

M_t is $T = \{0, \alpha, \gamma, \alpha\beta, \beta\alpha, \gamma\beta, \alpha\beta\beta\}$. $T - \{0\}$ is a D-class, union of two R-classes (and three L-classes) which is represented in Fig. 5.

For example, $I = \{0, \alpha, \alpha\beta, \alpha\beta\beta\}$ is a 0-minimal right ideal of M_t and $\mathscr{B}$ is the automaton shown in Fig. 6. If we choose $q_0 = 1$ then φ is defined by

$$\alpha\varphi = q_0\alpha = 1, \qquad \alpha\beta\varphi = q_0\alpha\beta = 3, \qquad \alpha\beta\beta\varphi = q_0\alpha\beta\beta = 2.$$

Now, $\alpha \sim_{M_t - \{0\}} \alpha\beta\beta$, so $\mathscr{B}_0$ has only two states, as shown in Fig. 7. And it is the minimal ida associated to R. We can see that $R = F((a, bc^*b)^*)$.

We now give another way to build this minimal automaton. Let $\mathscr{A} = (Q, F, Q, Q)$ an ida such that $\mathscr{A}^* = R$. For every $q \in Q$, let L_q be the set of words which are the label of some path in $\mathscr{A}$, starting in q. We consider the equivalence relation in Q: $q \sim q'$ iff $L_q = L'_q$. And $\hat{\mathscr{A}}$ is the quotient of $\mathscr{A}$ for this relation: $\hat{Q} = Q/\sim$ and $\hat{F}$ is defined by

$$(p, a, p') \in \hat{F} \text{ iff } \exists q \in p\ \exists q' \in p'\ (q, a, q') \in F.$$

Obviously, $\hat{\mathscr{A}}$ is an ida such that $\mathscr{A}^* = R$. $\hat{\mathscr{A}}$ is said to be reduced for $\sim$ ($\sim$ is the identity in $\hat{\mathscr{A}}$). And the canonical mapping $Q \to \hat{Q}$ is a morphism from $\mathscr{A}$ to $\hat{\mathscr{A}}$.

Proposition 3.4. *Let $\mathscr{A}$ be an ida recognizing an FTR-language R. Then $\hat{\mathscr{A}}$ is the minimal ida of R (up to an isomorphism).*

Proof. It suffices to prove that, in the previous construction, if $\mathscr{A}$ is a reduced ida, then the morphism $\hat{\psi}: \mathscr{A} \to \mathscr{B}_0$ is an isomorphism. Let q and q' be such that $q\hat{\psi} = q'\hat{\psi}$. There exist r and $r' \in I - \{0\}$ such that $q_0 r = q$, $q_0 r' = q'$ and $r\psi = r'\psi$. That means that

$$\forall u, v \quad urv \in M_t - \{0\} \text{ iff } ur'v \in M_t - \{0\}.$$

Let $y \in L_q$. So $q \cdot y = p$ and $p \neq s$. Therefore, $q_0 r(y\theta) = p$ and $r(y\theta) \neq 0$. Since $r\psi = r'\psi$, $r'(y\theta) \neq 0$ and so $q_0 r'(y\theta) \neq s$. But $q_0 r'(y\theta) = q'(y\theta)$, so $y \in L_{q'}$. And $L_q \subset L_{q'}$. $L_{q'} \subset L_q$ is proved in a symmetrical way. So $L_q = L_{q'}$ and then $q = q'$ since $\mathscr{A}$ is reduced. Whence $\hat{\psi}$ is an isomorphism. □

Example 3.5. We look again to the automaton $\mathscr{A}$ of Example 3.3. We have $L_1 = L_2$. So $\hat{\mathscr{A}}$ is the one shown in Fig. 8, and $\hat{\mathscr{A}}$ is isomorphic to $\mathscr{B}_0$.

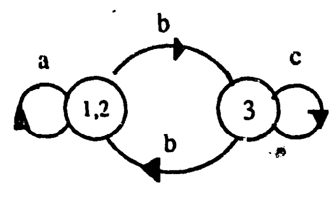


Fig. 8.

At last, we shall give the relation between the minimal ida recognizing an FTR-language and its usual minimal deterministic automaton [4] (with a single initial state).

Definition. Let $\mathscr{A} = (Q, F)$ be a finite automaton. $\mathscr{B} = (Q', F')$ is a terminal subautomaton of $\mathscr{A}$ iff

- $\mathscr{B}$ is a subautomaton of $\mathscr{A}$,
- $\forall q \in Q' \, \forall a \in A \, \forall q' \in Q \; (q, a, q') \in F' \Rightarrow q' \in Q'$.

Proposition 3.6. *Let $\mathscr{A}_0$ be the usual minimal deterministic automaton recognizing an FTR-language R. Then, the minimal ida recognizing R is a terminal subautomaton of $\mathscr{A}_0$.*

Proof. It is quite clear from Proposition 3.4. Let $\mathscr{A}_0 = (Q, F, q_0, Q)$ be the minimal deterministic automation with a single initial state such that $\mathscr{A}^* = R$. Since R is an FTR-language, $\mathscr{A}_0$ has the following property:

$$\forall q \in Q \, \exists a \in A \, \exists q' \in Q \quad (q, a, q') \in F.$$

So, among the strongly connected components of $\mathscr{A}_0$, there exists at least one which is a terminal subautomaton of $\mathscr{A}_0$. Let $\mathscr{B}_0$ be such a strongly connected component. $\mathscr{B}_0 = (Q', F', Q', Q')$. $\mathscr{B}_0$ is reduced: Suppose $L_q(\mathscr{B}_0) = L_{q'}(\mathscr{B}_0)$ for some $q, q' \in Q'$. $L_q(\mathscr{B}_0) = L_q(\mathscr{A}_0)$ and $L_{q'}(\mathscr{B}_0) = L_{q'}(\mathscr{A}_0)$ ($\mathscr{B}_0$ is a terminal subautomaton of $\mathscr{A}_0$). And $L_q(\mathscr{A}_0) = L_{q'}(\mathscr{A}_0)$ implies $q = q'$ ($\mathscr{A}_0$ is reduced).

$\mathscr{B}_0^* = \mathscr{A}_0^*$: The inclusion $\mathscr{B}_0^* \subset \mathscr{A}_0^*$ is obvious. Let $u \in \mathscr{A}_0^*$. Consider a state $q \in Q'$, and a word v such that $q_0 \rightarrow_v q$ in $\mathscr{A}_0$. R is transitive, so there exists $w \in A^*$ such that $vwu \in R$. So there exists a path $q_0 \rightarrow_v q \rightarrow_{wu} q'$ and the states of the path $q \rightarrow_{wu} q'$ belong to Q' because $\mathscr{B}_0$ is a terminal subautomaton of $\mathscr{A}_0$. So $u \in \mathscr{B}_0^*$. □

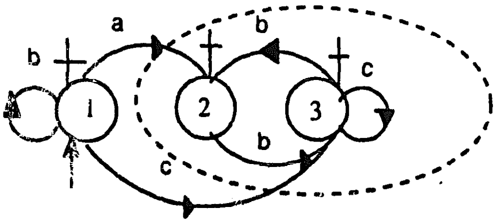


Fig. 9.

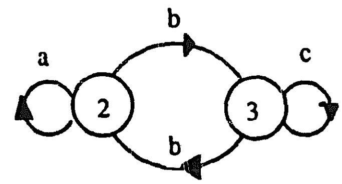


Fig. 10.

Example 3.7. The usual minimal deterministic automaton recognizing $R = F((a, bc^*b)^*)$ is as shown in Fig. 9. And the terminal strongly connected component of $\mathscr{A}_0^*$ is the minimal ida of R, as shown in Fig. 10. In this example, $\mathscr{A}_0$ has only one terminal strongly connected component; it is not a particular case as is proved below.

Corollary 3.8. *The usual minimal deterministic automaton of an FTR-language has only one terminal strongly connected component.*

Proof. Suppose there exists at least two terminal strongly connected components in $\mathscr{A}_0^*$: $\mathscr{B}$ and $\mathscr{B}'$. By Proposition 3.6, they are isomorphic. Let q and q' be two different states of $\mathscr{B}$ and $\mathscr{B}_0$ respectively, associated in this isomorphism. We have $L_q(\mathscr{A}_0) = L_q(\mathscr{B}) = L_{q'}(\mathscr{B}') = L_{q'}(\mathscr{A}_0)$. So, $\mathscr{A}_0$ is not reduced and there is a contradiction. □

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